An Approximate Method for Computing Blocking Probability in Switching Networks

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Abstract—**This paper describes an approximate method for evaluating the blocking probability of a switching network. The model used is C. Y. Lee's probability linear graph. By changing the structure of the graph between a pair of adjacent stages in two ways, it is possible to obtain upper and lower bounds for the blocking probability. The calculation of the bounds is performed on graphs which have a simpler structure as a result of the changes made. A suitable form is then chosen for the approximate blocking probability within the bounds. Typical multistage networks are treated in this fashion, and the approximate results are compared with the accurate blocking probabilities. The method is simple and inexpensive and produces reasonably accurate results.**

I. Introduction

IN A TELEPHONE switching network [1], [2], for reasons of economy, it is not possible to provide a connecting path between every pair of subscribers. Consequently, a demand for service may be unsatisfied when all the paths capable of providing this service in the switching network are busy. The probability that all the paths between a pair of terminals in the network are busy is known as the network blocking probability. The network blocking probability for an existing network can be measured directly by accumulating sufficient statistics. However, when a network is being designed one must be able to calculate the blocking probability. Thus, it is desirable to have an analytical model for estimating the blocking probability of a given switching network.

Perhaps the most widely used analytical model for evaluating the blocking probability of a given switching network is that of C. Y. Lee [3]. In his model, blocking probability is easily computable when the network consists of a series-parallel arrangement of links. However, in more complicated (and more realistic) networks, the computation may be very involved. For this reason, Grantges and Sinowitz [4] have proposed a simulation method to avoid the computational difficulties associated with complex networks. But the problems with the simulation are as follows: It simulates only an approximated model; it provides only numerical results and hence lacks the insight of an analytical method; and it usually requires a large amount of programming effort.

Therefore, it is convenient to have an approximate formula for estimating complex network blocking probability readily. This will not only enable us to evaluate whether simulation or complex computational effort is worth spending on a given switching network, but also will provide a tool for checking the results from computation or simulation. Indeed, for some purposes, blocking probability of a given network obtained by means of an easily computable approximate formula is sufficient.

Of course, if the switching network is relatively simple, the blocking probability can be easily calculated. Hence, in this case, there is no need for simulation or approximation. Therefore, the approximate formula developed in the following will be useful only when the switching network has four or more stages and is not a simple series-parallel structure.

II. Basic Model

We will consider a general L-stage switching network, as shown in Fig. 1. It is assumed that there is always one and only one junctor between any pair of switches in adjacent

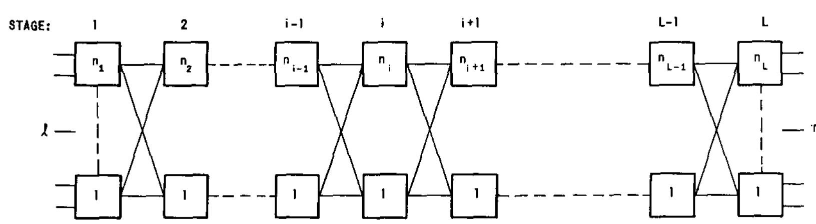

Fig. 1. General switching network.

stages. There are no junctors between switches within a stage nor between switches which are not in adjacent stages. It should be noted that when there are more than one junctors between a pair of switches in the adjacent stages, this model is still applicable if these multiple junctors are reduced to an equivalent single junctor. Therefore,

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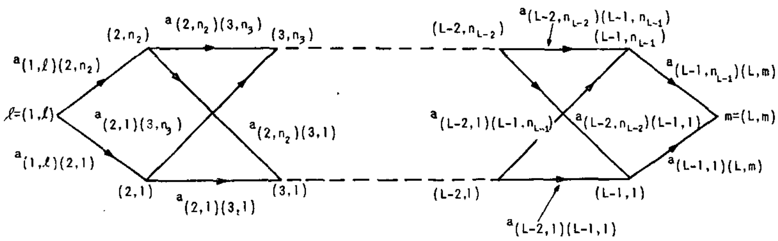


Fig. 2. Linear graph for the network.

between any input terminal l and any output terminal m, a linear graph [3] can be drawn, as shown in Fig. 2. In the linear graph, each switch is represented by a node and each junctor by a directed link. The notation used is as follows:

(i,j) is the jth node in the ith stage of the switching network,

n_i is the number of nodes (switches) in stage i, and

$a_{(i,j)(i+1,k)}$ is the link occupancy of the link between nodes (i,j) and $(i+1,k)$, i.e., the probability that the link between nodes (i,j) and $(i+1,k)$ is busy.

The general formula for calculating the blocking probability B between a pair of nodes l and m in a switching network is as follows [3]. Let E be the event that there is a path through the linear graph (the network), $E_{(i,j)(i+1,k)}$ be the event that the link between nodes (i,j) and $(i+1,k)$ is idle, and A_h be the event that path h between terminals l and m is idle. Then

$$B = 1 - Pr\{E\}$$
$$= 1 - Pr\left\{\bigcup_h A_h\right\}$$

with

$$A_h = \bigcap_{i=1}^{L-1} E_{(i,j)(i+1,k)} \qquad (1)$$

where the links joining together form path h.

The blocking probability B may also be found by using its generating function [3]. Suppose that there are M directed paths in the linear graph. Each directed path is composed of links

$$X_{(1,l)(2,i_1)},\ X_{(2,i_1)(3,i_2)},\ \ldots,\ X_{(L-1,i_{L-2})(L,m)}$$

in series. Let us agree to denote the directed path by the formal product f_i, i.e.,

$$f_i = X_{(1,l)(2,i_1)} \cdot X_{(2,i_1)(3,i_2)} \ldots X_{(L-1,i_{L-2})(L,m)}. \qquad (2)$$

In this formal product, the X's are considered as undefined real numbers and are manipulated as such except for the reduction rule

$$X_{(i,j)(i+1,k)} \cdot X_{(i',j')(i'+1,k')} = X_{(i,j)(i+1,k)} \qquad (3)$$

if $i = i'$, $j = j'$, and $k = k'$. Otherwise the operation is ordinary multiplication. Then, the generating function $B(t)$ of the blocking probability B is defined as

$$B(t) = \prod_i{}^* (1 - f_i t) \qquad (4)$$

where Π^* denotes formal product such that the reduction rule is always carried out. After $B(t)$ is expanded into a sum of products, $X_{(ij,)(i+1,k)}$ is replaced by $(1 - a_{(i,j)(i+1,k)})$. Then the blocking probability is equal to the generating function with t unity, i.e., $B = B(1)$.

In the case when the occupancies of the links between two adjacent stages are identical, that is

$$a_{(i,1)(i+1,1)} = a_{(i,1)(i+1,2)} = \ldots\ldots = a_{(i,n_i)(i+1,n_{i+1})}$$

but $a_{(i,j)(i+1,k)}$ is not necessarily equal to $a_{(i',j')(i'+1,k')}$; the blocking probability B sometimes can be found more easily by means of combinatorial analysis [5] than by means of the above two methods. However, when the network is relatively complex, even in this special case (when all link occupancies are the same), computational difficulty still arises.

Therefore, we propose to obtain a general approximate formula for this model.

III. Derivation of Approximate Formula

A. The Blocking Probability Bounds

We shall pick any two adjacent stages, i and $i+1$, from the linear graph of the general L-stage switching network, as shown in Fig. 3. We are going to find an upper and a lower bound of the network blocking probability by disturbing only this section.

1. Lower Bound: In our general L-stage switching network, there are no junctors between switches within a stage. Therefore, there is no link between nodes within a stage in the section (Fig. 3) considered. Since the blocking probability of the switching network will be calculated in accordance with C. Y. Lee's model, it will not be affected by the addition of extra links whose occupancy is unity. Suppose, to each stage in the section considered, we add extra links between switches within each stage. Then the resulting network blocking probability will be smaller than our general network blocking probability if the occupancies of these extra links are not all unity. This can be easily seen from the generating function (4). It is convenient to let all the occupancies of these extra links be zero; then the network section considered (Fig. 3) takes the form of Fig. 4. For the entire network with only section $(i,i+1)$ designed in this way, the network structure is simplified, and the network blocking probability will be smaller than that of the actual general network.

2. Upper Bound: On the other hand, if we assume that the occupancies of some junctors of our general L-stage

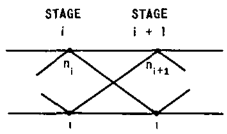


Fig. 3. A section of the network.

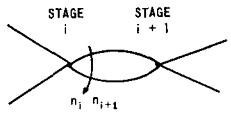


Fig. 4. Modification for evaluating B_0.

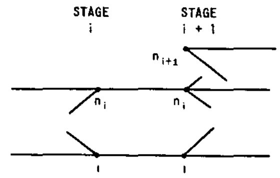


Fig. 5. Modification for evaluating B_1.

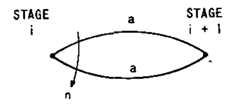


Fig. 6. Model for $F_{i,\, i+1}$.

switching network are increased, the resulting blocking probability will obviously be greater. Again, consider the section of adjacent stages i and $i+1$ (Fig. 3) and assume $n_i \leq n_{i+1}$. If occupancies of all the links except $a_{(i,j)(i+1,j)}$'s are equated to unity, the section considered will then take the form of Fig. 5. Thus the network structure with only section $(i,i+1)$ designed in this way is simplified, and the resulting network blocking probability will be greater than that of the actual network. It should be noted that there are many choices of retaining $a_{(i,j)(i,+1,j)}$'s for an upper bound network structure. One choice may give a better result than another. Usually, keeping the smaller occupancies will result in a better approximation.

Summary: To sum up, let

- B be the blocking probability between terminals l and m in our general L-stage switching network;
- B_0 be the blocking probability between the terminals l and m with the occupancies of the additional inserted junctors within stage i and those within stage $i+1$ all zero; and
- B_1 be the blocking probability between terminals l and m when all link occupancies $a_{(i,j)(i+1,k)}$'s except $j = k$ between adjacent stages i and $i+1$ are equated to unity; then, we have $B_0 \leq B \leq B_1$.

B. Approximate Formula

In order to get a simple (but a nontrivial) approximation, assume that the approximate blocking probability B_a is of the form:

$$B_a = B_0 + (B_1 - B_0)F_{i,i+1}, \tag{5}$$

where the value of $F_{i,i+1}$ must be in the range of $0 \leq F_{i,i+1} \leq 1$ and $F_{i,i+1}$ depends on the actual occupancies and the number of links between stages i and $i+1$. To determine a suitable factor $F_{i,i+1}$ for a given switching network, let us consider the following facts:

1) As the link occupancies $a_{(i,j)(i+1,k)}$'s between stages i and $i+1$ approach zero, the network blocking probability B approaches B_0. Hence, the value of $F_{i,i+1}$ should decrease as the link occupancies decrease, and should reach zero when the occupancies become zero.

2) Equating a link occupancy to unity is equivalent to removing that link from the network. The larger the number of links between stages i and $i+1$, the larger the number of links removed in order to obtain B_1. Hence, the larger the number of links, the larger the difference $B_1 - B$. Therefore, to achieve a better approximation with our formula (5), $F_{i,i+1}$ should decrease as the number of links between stages i and $i+1$ increases.

To satisfy the above two conditions, many functions can be constructed. However, we must remember that the approximate formula must be simple or else it will lose its significance. Suppose that the factor $F_{i,i+1}$ will take the form analogous to the blocking probability of a simple network which has only two nodes connected by n links, as shown in Fig. 6. This blocking probability is a^n (where a is the occupancy of each link) and thus satisfies our conditions. Then to account for condition 1) we propose that the occupancy of every link in Fig. 6 be proportional to the arithmetic average of the occupancies of the links between the two adjacent stages i and $i+1$ considered. To account for condition 2), we propose that the number of links in Fig. 6 equals the geometric average of the number of links per node of the two adjacent stages i and $i+1$.

The arithmetic average of the occupancies of the links between the two adjacent stages i and $i+1$ is defined as usual, i.e.:

$$\bar{a}_{i,i+1} = \frac{\sum_{j=1}^{n_i} \sum_{k=1}^{n_{i+1}} a_{(i,j)(i+1,k)}}{n_i n_{i+1}}. \tag{6}$$

The occupancy of a link in Fig. 6 is proportional to $\bar{a}_{i,i+1}$. When the occupancies of the links between stages i and $i+1$ are identical to those for obtaining B_1, the value of $F_{i,i+1}$ must be unity. Therefore, the occupancy of a link in Fig. 6 is $\bar{a}_{i,i+1}/\bar{a}'_{i,i+1}$ where, assuming $n_i \leq n_{i+1}$,

$$\bar{a}'_{i,i+1} = \frac{n_i\, n_{i+1} - n_i + \sum_{j=1}^{n_i} a_{(i,j)(i+1,j)}}{n_i\, n_{i+1}} \tag{7}$$

which is the arithmetic average of the occupancies of the links between stages i and $i+1$ of the linear graph for evaluating B_1.

Also, as usual, the geometric average of the number of links between stages i and $i+1$ per node is $\sqrt{n_i n_{i+1}}$. Therefore, in Fig. 6, each link has an occupancy of $a_{i,i+1}/\bar{a}'_{i,i+1}$, and there are $\sqrt{n_i n_{i+1}}$ links. Thus, its blocking probability and hence, the value of factor $F_{i,i+1}$ is

$$F_{i,i+1} = \left(\frac{\bar{a}_{i\,i+1}}{\bar{a}'_{i,i+1}}\right)^{\sqrt{n_i n_i+1}} \tag{8}$$

which satisfies our requirements.

C. Choice of Starting Point

The result of the approximate blocking probability between an input and an output terminal is not unique. It depends not only on the choice of retaining $a_{(i,j)(i+1,j)}$'s for obtaining the upper bound network structure as pointed out previously, but it also depends on the choice of the two adjacent stages to be disturbed for obtaining the approximation.

In retaining $a_{(i,j)(i+1,j)}$'s for the upper bound structure, it has been pointed out that better approximation results by keeping the smaller occupancies. For the choice of the two adjacent stages to be disturbed for obtaining the approximation, the best result occurs when $(B_1 - B_0)$ is a minimum. Since $(B_1 - B_0)$ is not known until it is computed, we suggest to use those stages which result in the greatest simplification. As a general rule, this may be achieved by disturbing the two adjacent stages having the largest number of actual links. This will be illustrated in the next section.

IV. Application of Approximate Formula

A. Four-Stage Switching Network

The linear graph between terminal $(1,l)$ and $(4,m)$ in a four-stage switching network is as shown in Fig. 7(a). The largest number of links in the linear-graph occurs between stages 2 and 3. Therefore, we are going to disturb these stages. To evaluate B_0, we imagine that there are links between switches within stage 2 and within stage 3 and the occupancies of these links are all zero. Thus, the linear graph for obtaining B_0 is as shown in Fig. 7(b). The linear graph for evaluating B_1 is obtained by equating all $a_{(2,i)(3,j)}$'s except $i = j$ to unity, as shown in Fig. 7(c), where $n_2 \leq n_3$ is assumed. As the linear graphs of Figs. 7(b) and 7(c) involve only parallel and series form of directed paths, using C. Y. Lee's model, we can easily evaluate their blocking probabilities. Thus

$$B_0 = 1 - \left(1 - \prod_{i=1}^{n_2} a_{(1,l)(2,i)}\right)\left(1 - \prod_{i=1}^{n_2}\prod_{j=1}^{n_3} a_{(2,i)(3,j)}\right) \cdot \left(1 - \prod_{j=1}^{n_3} a_{(3,j)(4,m)}\right), \tag{9}$$

and

$$B_1 = \prod_{i=1}^{n_2} [1 - (1 - a_{(1,l)(2,i)}) (1 - a_{(2,i)(3,i)})(1 - a_{(3,i)(4,m)})]. \tag{10}$$

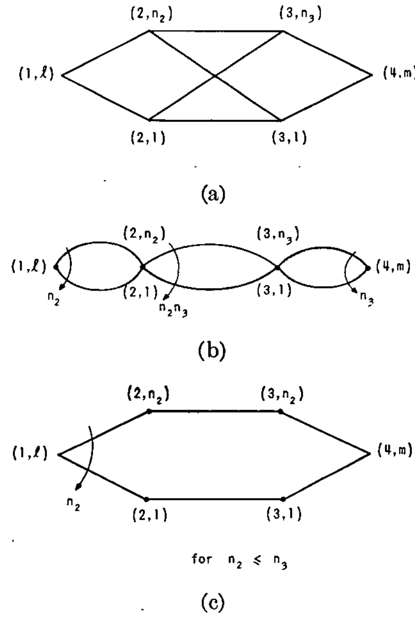


Fig. 7. (a) 4-stage network. (b) Graph for B_0. (c) Graph for B_1.

Replacing i by 2 in (6)–(8), we get

$$F_{2,3} = \left(\frac{\bar{a}_{2,3}}{\bar{a}'_{2,3}}\right)^{\sqrt{n_2 n_3}} \tag{11}$$

Hence, substituting (9)–(11) into (5) yields the approximate blocking probability.

B. Switching Network with More Than Four States

1. Step-by-Step Method: Let us consider a six-stage switching network having the linear graph shown in Fig. 8(a). We can disturb the occupancies of the links between stages 4 and 5 of the linear graph resulting in Figs. 8(b) and 8(c). The blocking probability between terminals $(1,l)$ and $(6,m)$ in Fig. 8(b) is B_0, and that in Fig. 8(c) is B_1 for that in Fig. 8(a). The $F_{i,i+1}$ factor here is, of course,

$$F_{4,5} = \left(\frac{\bar{a}_{4,5}}{\bar{a}'_{4,5}}\right)^{\sqrt{n_4 n_5}} \tag{12}$$

To find B_0, we may divide the linear graph in Fig. 8(b) into two subgraphs in series. One subgraph consists of stages 1 to 4, another consists of stages 4 to 6. If the blocking probability between terminals $(1,l)$ and $(4,1)$ and that between $(4,1)$ and $(6,m)$ are known, then the blocking probability between terminals $(1,l)$ and $(6,m)$ can be easily calculated in the standard manner. For the subgraph consisting of stages 4 to 6, the blocking probability is

$$\prod_{i=1}^{n_5} a_{(5,i)(6,m)} + \left(1 - \prod_{i=1}^{n_5} a_{(5,i)(6,m)}\right) \prod_{j=1}^{n_4}\prod_{k=1}^{n_5} a_{(4,j)(5,k)}.$$

For the subgraph consisting of stages 1 to 4, the blocking probability may be calculated in accordance with Section IV-A above.

To find B_1, we may let

$$a_{(4,i)(6,m)} = a_{(4,i)(5,i)} + (1 - a_{(4,i)(5,i)})\, a_{(5,i)(6,m)} \tag{13}$$

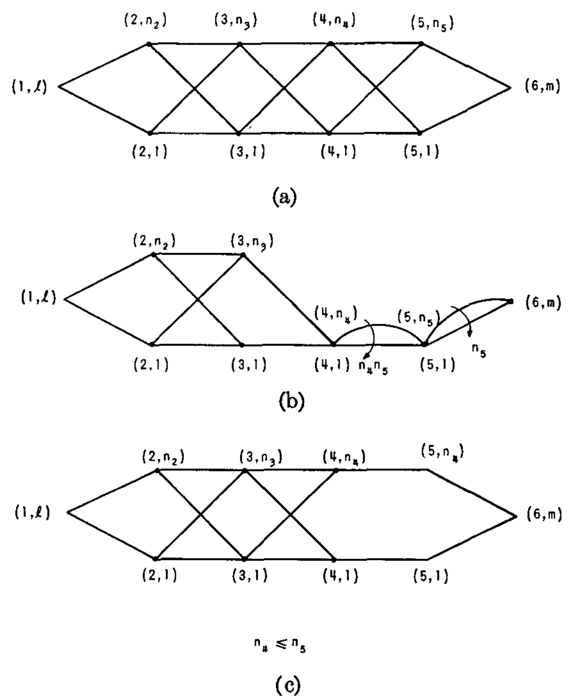


Fig. 8. (a) 6-stage network. (b) Graph for B_0. (c) Graph for B_1.

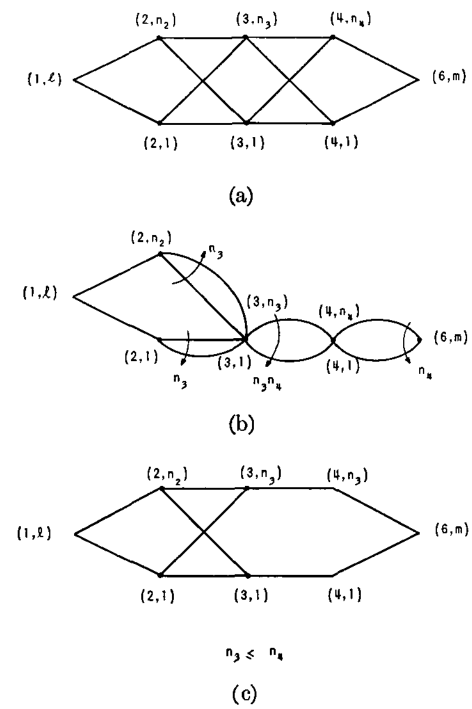


Fig. 9. Second step in simplifying the network of Fig. 8(a).

in Fig. 8(c). Thus, Fig. 8(c) becomes essentially a five-stage network shown in Fig. 9(a). In this linear graph [Fig. 9(a)], we can disturb another pair of adjacent stages resulting in Figs. 9(b) and 9(c). The blocking probability between terminals (1,l) and (6,m) in Fig. 9(b) and that in Fig. 9(c) are B_0 and B_1, respectively, for that in Fig. 9(a). Since the linear graph for Fig. 9(b) is a series-parallel type, the blocking probability can be easily calculated. In Fig. 9(c), if we further let

$$a_{(3,i)(6,m)} = a_{(3,i)(4,i)} + (1 - a_{(3,i)(4,i)})\, a_{(4,i)(6,m)} \quad (14)$$

it becomes a 4-stage network. Thus the blocking probability can be calculated in accordance with Section IV–A.

To summarize at this point: The blocking probability of 4-stage graph Fig. 9(c) is used as B_1 (together with B_0 obtained from Fig. 9(b) and with $F_{3,4}$) to calculate the blocking probability of the 5-stage network Fig. 9(a). In turn, this blocking probability is used as B_1 (together with B_0 obtained from Fig. 8(b) and with $F_{4,5}$) to calculate the blocking probability of the 6-stage network Fig. 8 (a).

While the above concerns only the six-stage network, this method of obtaining the blocking probability between two terminals by reducing one stage at a time may be extended to any number of stages. It should be noted that details of computation are affected by the choice of stages to be disturbed. Suppose, in our previously considered six stages, we disturb stages 3 and 4 instead of 4 and 5. We obtain the linear graphs of Fig. 10(b) and 10(c) for its B_0 and B_1, respectively. Fig. 10(b) is a series-parallel type of network, and hence its blocking probability can be easily found. Although the blocking probability of Fig. 10(c) cannot be easily evaluated in its present form, we can apply this step-by-step technique described above. In

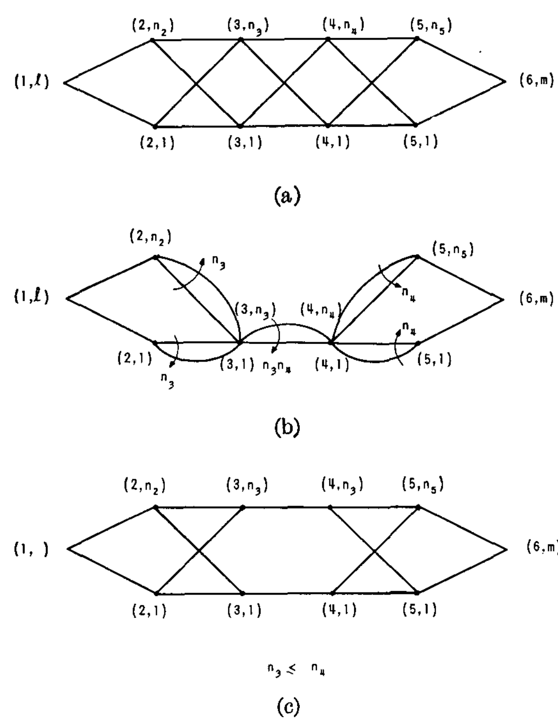


Fig. 10. A different reduction of the network of Fig. 8(a).

general, these two approximated blocking probabilities may not be the same; but hopefully both should be reasonably close to the exact figure.

2. Average F-Factor Method: When the number of stages in a switching network is large, the step-by-step method described above becomes very laborious. In this case, we may obtain the approximate blocking probability as follows: First, we redefine:

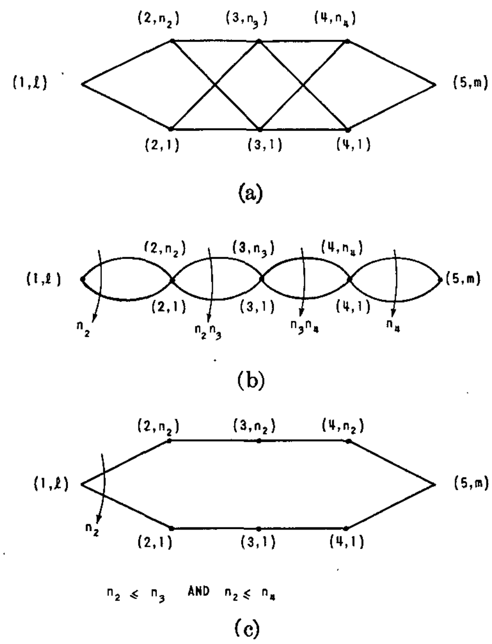


Fig. 11. Illustrating the average F-factor method.

B_0 as the blocking probability between the terminals l and m with the occupancies of the imaginary links between switches within each stage all zero.

B_1 as the blocking probability between terminals l and m when all link occupancies $a_{(i,j)(i+1,k)}$'s between stages i and $i+1$ except $j = k$ are equated to unity for $i = 2, 3, \ldots L\text{-}2$.

Next, we let

$$F = \frac{1}{L\text{-}3} \sum_{i=2}^{L-2} F_{i,i+1}, \quad (15)$$

where $F_{i,i+1}$ is the $F_{i,i+1}$ − factor of (8), that is

$$F_{i,i+1} = \left(\frac{\bar{a}_{i,i+1}}{\bar{a}'_{i,i+1}}\right)^{\sqrt{n_i n_{i+1}}}.$$

Finally, since the relation $B_0 \leq B \leq B_1$ still holds, similar to (5), we assume

$$B_a = B_0 + (B_1 - B_0)F. \quad (16)$$

As an example, consider a five-stage switching network having a linear graph as shown in Fig. 11(a). Then, the linear graphs for B_0 and B_1 become as shown in Figs. 11(b) and 11(c) respectively. Since these linear graphs involve only parallel and series links, we can easily evaluate the blocking probabilities. As have been noted before, the factor F is quite easy to compute. Hence, the blocking probability can be readily obtained. Thus a cruder approximation can be obtained with less effort.

V. Numerical Examples

We have seen that B_0, B_1, and $F_{i,i+1}$ − factor are relatively simple to compute. At this point, we are going to investigate how well our approximate formula is in agreement with results obtained by means of direct computation or simulation. When the switching network is relatively complex and all link occupancies are different, the blocking probability can be approximated quite easily with the method presented here. However, the exact blocking probability of such a network may not be easy to calculate. Therefore, we are going to compare the results by different methods only for some special cases in which the blocking probability can be directly calculated or has been obtained by simulation.

Case 1

Consider the network having a linear graph, as shown in Fig. 12(a), in which the occupancy of every link equals a. In this case, it is not difficult to find the blocking probability between terminals l and m by applying either (1) or (4). Thus:

$$B = 1 - 2(1-a)^2 - (1-a)^3 + 3(1-a)^4 - (1-a)^5. \quad (17)$$

If we are going to find the approximate blocking probability by the method presented here, we must first make Fig. 12(a) equivalent to the model we considered in Section II above, as shown in Fig. 12(b). Thus, it becomes a standard 4-stage switching network. Therefore, according to (9), (10), and (11), respectively, we get

$$B_0 = a^2(2 - a^2) \quad (18)$$

$$B_1 = a^2(2 - a)^2 \quad (19)$$

and

$$F_{2,3} = \left(\frac{1+a}{2}\right)^2. \quad (20)$$

Here we have retained $a_{(2,1)(3,1)} = 0 = a_{(2,2)(3,2)}$ and equated $a_{(2,1)(3,2)} = a$ and $a_{(2,2)(3,1)} = 1$ to unity for obtaining B_1. Then the approximate blocking probability is given by (5). The results of B and B_a are plotted for various values of a in Fig. 17.

Case 2

Consider the network having a linear graph of Fig. 13, where the occupancy of every link equals a. If we use (1) or (4) to find its blocking probability between terminals l and m, the process is very laborious. Since the occupancies of the links are identical, we can use the combinatorial analysis approach [5] to find its blocking probability. Thus,

$$B = \sum_{i=0}^{10} \binom{10}{i} a^{10-i} (1-a)^i \, [a + (1-a)a^i]^{10}. \quad (21)$$

The approximate blocking probability is given by (5), where

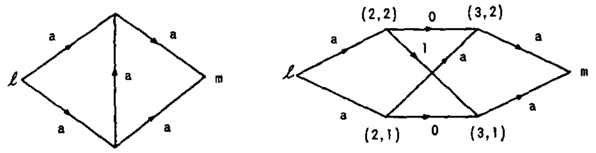
$$B_0 = 1 - (1 - a^{10})^2 (1 - a^{100}) \quad (22)$$

$$B_1 = [1 - (1-a)^3]^{10} \quad (23)$$

and

$$F_{2,3} = \left(\frac{10a}{9+a}\right)^{10}. \quad (24)$$

The results of both B and B_a are plotted in Fig. 18.



(a) (b)

Fig. 12. Network for Case 1.

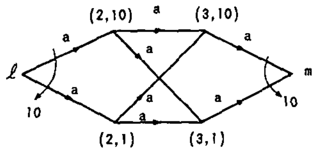


Fig. 13. Network for Case 2.

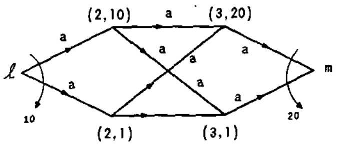


Fig. 14. Network for Case 3.

Case 3

Consider a 4-stage switching network linear graph shown in Fig. 14, where the number of nodes in stages 2 and 3 are not equal. As the occupancies of all links are equal, we can still make use of the combinatorial analysis approach which gives:

$$B = \sum_{i=0}^{10} \binom{10}{i} a^{10-i}(1-a)^i\,[a + (1-a)a^i]^{20}. \quad (25)$$

The approximate blocking probability B_a is, of course, (5) with

$$B_0 = 1 - (1-a^{10})(1-a^{200})(1-a^{20}), \quad (26)$$

$$B_1 = [1 - (1-a)^3]^{10}, \quad (27)$$

and

$$F_{2,3} = \left(\frac{20a}{19+a}\right)^{\sqrt{200}}. \quad (28)$$

The results of both B and B_a are plotted in Fig. 19.

Case 4

Grantges and Sinowitz have studied a linear graph of moderate complexity, shown in Fig. 15. Their simulated results and the calculated blocking probabilities for $0.44 \leq a \leq 0.58$ are shown in Fig. 20. For comparison, in Fig. 20 we also plot the approximate blocking probability B_a by the method presented here. Again B_a is (5) with

$$B_0 = 1 - (1-a^8)^2\,(1-a^{32}), \quad (29)$$

$$B_1 = [1 - (1-a)^3]^8, \quad (30)$$

and

$$F_{2,3} = \left(\frac{4(1+a)}{7+a}\right)^8. \quad (31)$$

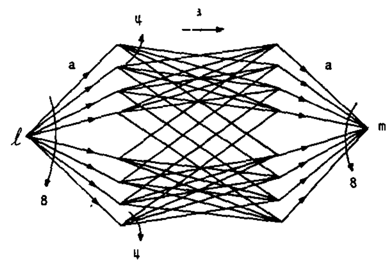


Fig. 15. Network for Case 4.

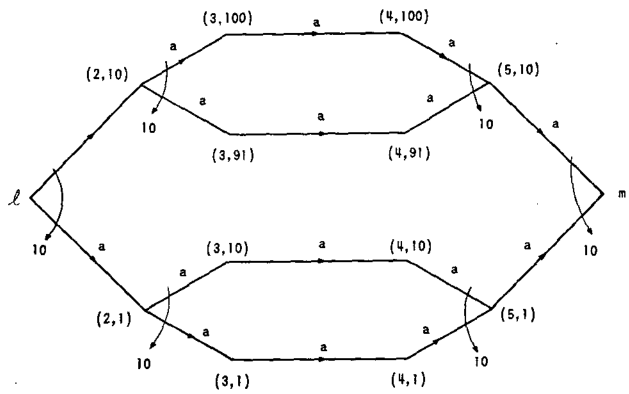


Fig. 16. Network for Case 5.

Case 5

For a 6-stage network having a linear graph of Fig. 16, the blocking probability between terminals l and m can be easily derived by means of the generating function (4). Thus,

$$B = \{1 - (1-a)^2\,[1 - (1 - (1-a)^3)^{10}]\}^{10}. \quad (32)$$

Its approximate blocking probability can be found in either one of the two ways: namely, step-by-step method and the average F-factor method. Since the latter is much easier to use, we are going to approximate the blocking probability by the average F-factor method only. We have

$$F_{2,3} = F_{4,5} = \left(\frac{90+10a}{99+a}\right)^{\sqrt{1000}} \quad (33)$$

$$F_{3,4} = \left(\frac{990+10a}{999+a}\right)^{100} \quad (34)$$

$$F = \frac{2F_{2,3} + F_{3,4}}{3} \quad (35)$$

$$B_0 = 1 - (1-a^{10})^2\,(1-a^{100})^3 \quad (36)$$

$$B_1 = [1 - (1-a)^5]^{10} \quad (37)$$

and

$$B_a = B_0 + (B_1 - B_0)F. \quad (38)$$

Both B and B_a have been plotted, as shown in Fig. 21.

By examining the results (Figs. 17–21) of the above cases, it is seen that the characteristics of the approximate results are generally in agreement with those of the accu-

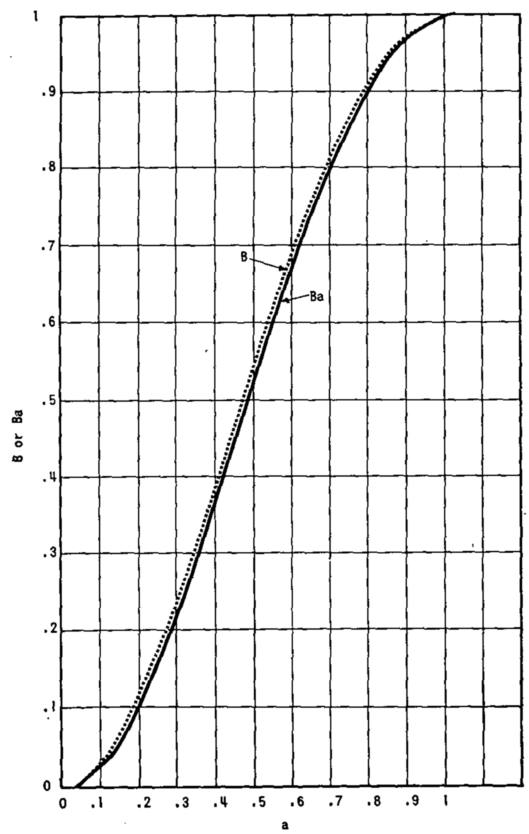


Fig. 17. Results for Case 1.

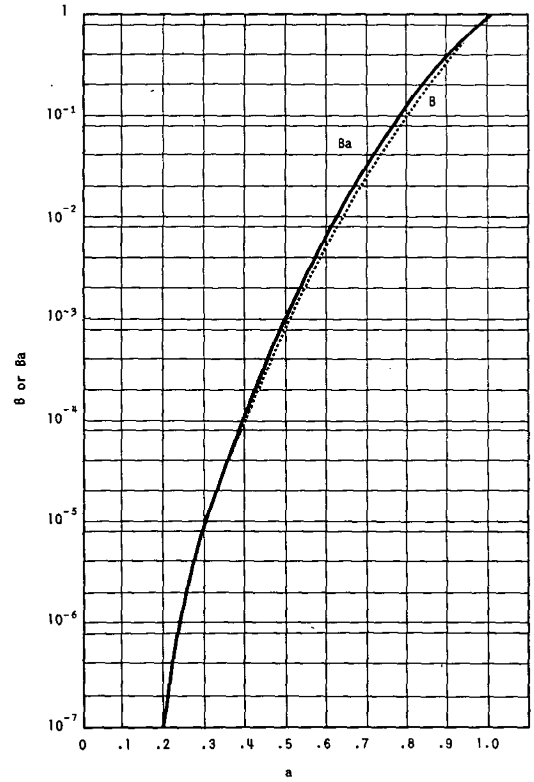


Fig. 19. Results for Case 3.

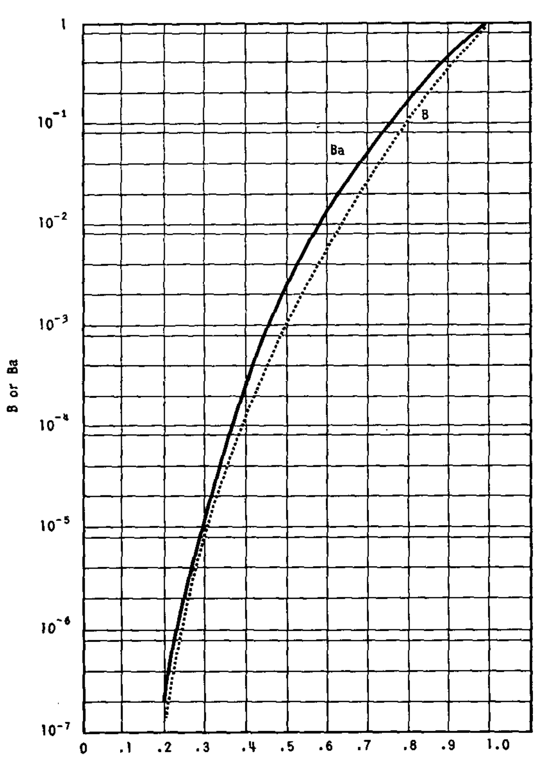


Fig. 18. Results for Case 2.

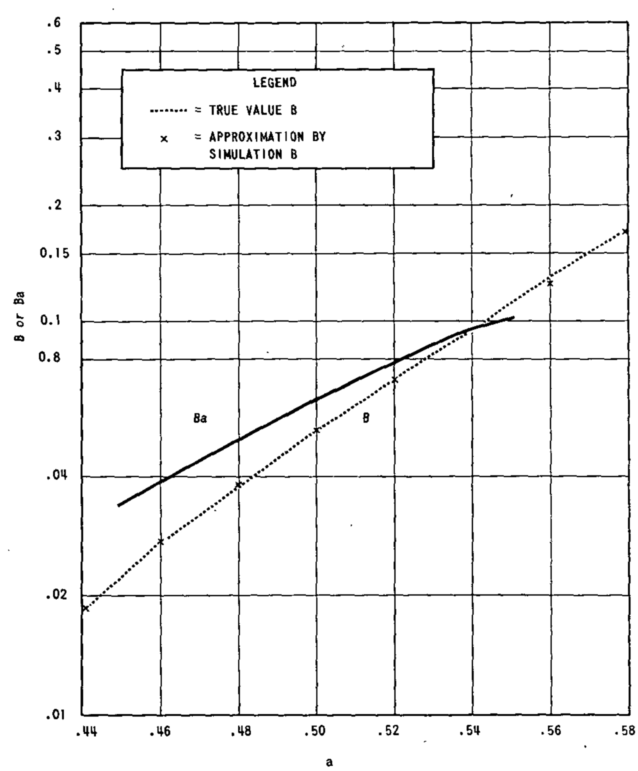


Fig. 20. Results for Case 4.

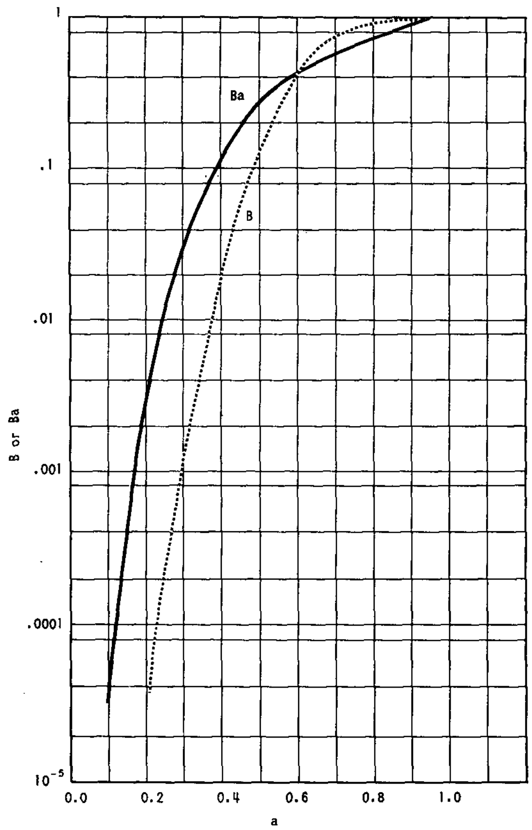


Fig. 21. Results for Case 5.

rate results. A more complete evaluation of the approximation is made difficult by the fact that the exact answer is not easily available. At any rate the approximation appears sufficiently good to be used to obtain a preliminary evaluation of the network.

VI. Conclusion

The approximate method proposed here for evaluating switching network blocking probability has been shown to produce reasonable results and has the advantage of being simple and inexpensive. In complex switching networks, it may serve as a guide to determine whether the effort of direct computation or simulation is worthwhile for a given network, and also provides a means of checking the results obtained from computation and simulation.

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